

Superaligned Beta Decay: Probing the Standard Model with Nuclei Near $N = Z$

J.C. Hardy¹ and I.S. Towner^{1,2}

¹ *Cyclotron Institute, Texas A & M University, College Station, TX 77843, U.S.A.*

² *Physics Department, Queen's University, Kingston, Ontario K7L 3N6, Canada*

Abstract

The decay of nuclei near $N = Z$ provides a sensitive method for probing the limitations of the Electroweak Standard Model. To date, the strengths of superallowed $0^+ \rightarrow 0^+$ beta-decay transitions have been determined with high precision from nine different short-lived nuclei, ranging from ^{10}C to ^{54}Co . Each result leads to an independent measure for the vector coupling constant, G_V , and collectively the nine values can be used to test the conservation of the weak vector current (CVC). The results support CVC to better than a few parts in 10,000 – a clear success for the Standard Model! However, when the average value of G_V as determined in this way, is combined with data from decays of the muon and kaon to test another prediction of the Standard Model, the result is much more provocative. A test of the unitarity of the Cabibbo-Kobayashi-Maskawa matrix fails by more than two standard deviations. This result, if confirmed by even more precise measurements in the future, could have far-reaching consequences.

This talk describes the current status of the nuclear measurements and compares the results with neutron-decay measurements, which are less precise but consistent with the nuclear results. New nuclear measurements aimed at achieving a definitive result will focus on even-even $N = Z - 2$ nuclei lighter than $A = 40$, and on odd-odd $N = Z$ nuclei heavier than $A = 60$.

1 Introduction

Superaligned $0^+ \rightarrow 0^+$ nuclear beta decays provide both the best test of the Conserved Vector Current (CVC) hypothesis and, together with the muon lifetime, the most accurate value for the up-down quark-mixing matrix element, V_{ud} , of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. This matrix should be unitary, and experimental verification of that expectation constitutes an important test of the Standard Model. With current world data for $0^+ \rightarrow 0^+$ beta decays [1] used to obtain a value for V_{ud} , and the standard values [2] taken for the other required elements of the CKM matrix, the unitarity test from the sum of the squares of the elements in the first row fails to meet unity by more than twice the estimated uncertainty. This result is tantalizingly close to establishing a definitive disagreement with the Standard Model, and prompts renewed efforts to improve the precision with which the test can be made. The challenges in doing so are both experimental and theoretical.

The nine $0^+ \rightarrow 0^+$ transitions that currently comprise the nuclear input data for V_{ud} are all between $T = 1$ analog states. Thus, nuclear-structure effects only enter at the level of differences between the parent and daughter wave functions. These differences, which are caused by Coulomb and charge-dependent nuclear forces, turn out to be very small, and introduce a correction, denoted

δ_C , of order 1% when the experimental ft -values are used to extract a value for the effective weak vector coupling constant, G'_V . Even a conservative estimate of the uncertainties in this correction indicates that structure-dependent uncertainties should not afflict the experimental determination of G'_V above the level of approximately $\pm 0.1\%$. To date, ft -value measurements have aimed at achieving this level of experimental precision or slightly better. Future improvements will depend not just upon higher precision being achieved in the measurements but also upon improvements – or increased confidence – in the calculated values of δ_C .

To calculate δ_C for a particular superallowed transition, it is important to have a reliable nuclear model that demonstrably fits nuclear properties in the same mass region. To refine the model's effectiveness in calculating charge-dependent effects, it is also valuable to have accurate experimental data on the c coefficient of the isobaric-multiplet mass equation (IMME) for the same 0^+ multiplet and, if possible, to have data on other, non-analog $0^+ \rightarrow 0^+$ decays from the same parent state. The nine superallowed $0^+ \rightarrow 0^+$ decays currently known with high precision occur among $N \simeq Z$ nuclei with $10 \leq A \leq 54$, where nuclear structure information is relatively abundant and reliable models exist. Any future improvements in δ_C are likely to come from measurements of additional superallowed transitions, especially those for which δ_C is anticipated to be particularly large. Such measurements will then constitute a test of the δ_C calculations and, if successful, will give a better indication of the uncertainty that should be applied to δ_C in the cases currently known, where the correction itself is smaller. All these future measurements will, of necessity, be among $N \simeq Z$ nuclei up to $A = 98$: they constitute one of the most demanding tasks we face in any branch of nuclear physics, with a requirement for high precision and for a thorough knowledge of the neighboring nuclear structure, sometimes in regions of very exotic nuclei.

This paper will outline the current status of the nuclear measurements bearing on V_{ud} , possible explanations for the disagreement with CKM unitarity, and future directions for study.

2 Current status of world data

Because the axial current cannot contribute in lowest order to transitions between spin-0 states, the experimental ft -value for a $0^+ \rightarrow 0^+$ transition is related directly to the vector coupling constant. Specifically, for an isospin-1 multiplet,

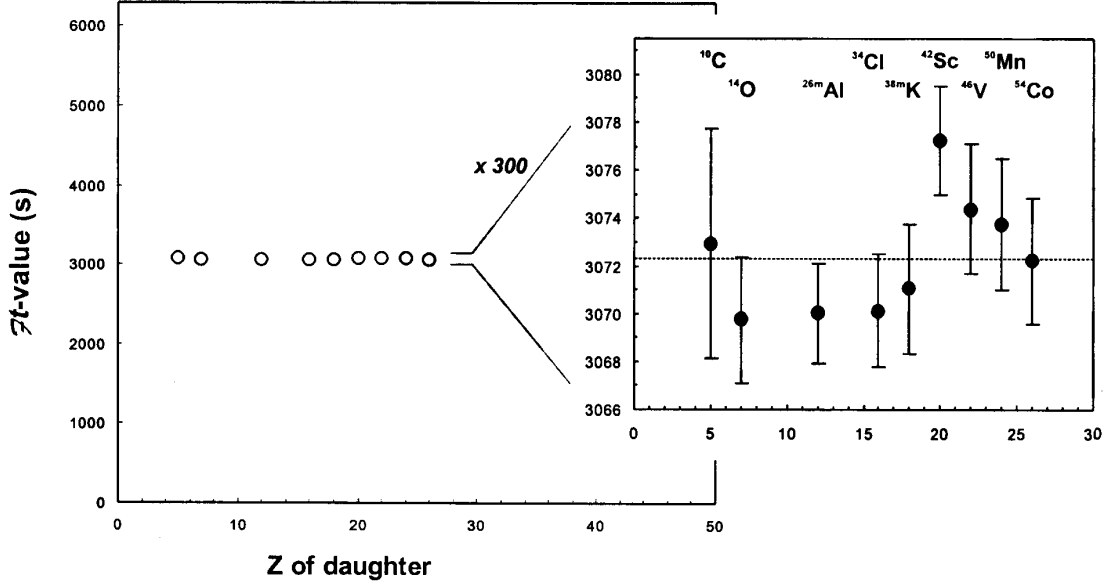
$$ft(1 + \delta_R) = \frac{K}{G_V'^2 \langle M_V \rangle^2}, \quad (1)$$

with

$$\begin{aligned} G_V' &= G_V(1 + \Delta_R^V)^{1/2}, \\ \langle M_V \rangle^2 &= 2(1 - \delta_C), \\ K/(\hbar c)^6 &= 2\pi^3 \hbar \ln 2 / (m_e c^2)^5 = (8120.271 \pm 0.012) \times 10^{-10} \text{GeV}^{-4} \text{s}, \end{aligned} \quad (2)$$

where f is the statistical rate function, t is the partial half-life for the transition, $\langle M_V \rangle$ is the Fermi matrix element and G_V is the primitive vector coupling constant. The physical constants used to evaluate K were taken from the most recent Particle Data Group publication [2]. These equations

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$$\mathcal{F}t = 3072.3 \pm 0.9$$

$$\chi^2/\nu = 1.1$$

Figure 1: Measured $\mathcal{F}t$ -values for the nine precisely measured $0^+ \rightarrow 0^+$ transitions. The results are consistent with a unique value for the vector coupling constant. The average $\mathcal{F}t$ -value is shown, together with the normalized chi-squared of the fit to the data.

also include three calculated correction terms – all of order 1%. We write δ_R as the nucleus-dependent part of the radiative correction, Δ_R^v as the nucleus-independent part of the radiative correction, and δ_C as the isospin symmetry-breaking correction. A general description of these three correction terms and the methods used in their calculation has appeared elsewhere [1, 3]. In the present context, it is sufficient to note that nuclear structure plays a small role in the determination of δ_R , but it is predominant for that of δ_C .

Equations (1) and (2) can now be combined into a form that is convenient for the analysis of experimental results:

$$\mathcal{F}t \equiv ft(1 + \delta_R)(1 - \delta_C) = \frac{K}{2G_V^2(1 + \Delta_R^v)}. \quad (3)$$

Here we have defined $\mathcal{F}t$ as the “corrected” ft -value. From this equation, it is evident that the $\mathcal{F}t$ -values obtained from $0^+ \rightarrow 0^+$ transitions in different nuclei can constitute a stringent test of CVC, which requires them all to be equal.

To date, superallowed $0^+ \rightarrow 0^+$ transitions have been measured to $\pm 0.1\%$ precision or better in the decays of nine nuclei, ^{10}C , ^{14}O , ^{26m}Al , ^{34}Cl , ^{38m}K , ^{42}Sc , ^{46}V , ^{50}Mn and ^{54}Co . World data

on Q -values, lifetimes and branching ratios – the results of over 100 independent measurements – were thoroughly surveyed [4] in 1989 and then updated several times since, most recently for the WEIN98 conference [1]. By applying the δ_R and δ_C corrections listed in reference [1], we converted the results into $\mathcal{F}t$ -values. These values are plotted in Fig. 1.

It is important to appreciate that the values of δ_R and δ_C each result from more than one independent calculation. In the case of δ_R , results come from a variety of primary sources, all in complete accord with one another; for δ_C , we have used an average of two independent calculations [5, 6] with assigned uncertainties that reflect the (small) scatter between them. Thus, in a real sense, both experimentally and theoretically, the $\mathcal{F}t$ -values illustrated in Fig. 1 represent the totality of current world knowledge. The uncertainties reflect the experimental uncertainties and an estimate of the *relative* theoretical uncertainties in δ_C . There is no statistically significant evidence of inconsistencies in the data ($\chi^2/\nu = 1.1$), thus verifying the expectation of CVC at the level of 3×10^{-4} , the fractional uncertainty quoted on the average $\mathcal{F}t$ -value: $\overline{\mathcal{F}t} = 3072.3 \pm 0.9$ s.

Now, in using this average $\mathcal{F}t$ -value to determine V_{ud} and test CKM unitarity, we must account for additional ± 1.1 s to account for the systematic difference between the two calculations of δ_C that we have combined in reaching this result. (For a more complete discussion of how we treat these theoretical uncertainties, see Ref. [4].) We now add the two errors linearly to obtain the value we use in subsequent analysis:

$$\overline{\mathcal{F}t} = 3072.3 \pm 2.0 \text{ s.} \quad (4)$$

The value of V_{ud} is obtained by relating the vector constant, G_v , determined from this $\overline{\mathcal{F}t}$ value, to the weak coupling constant from muon decay, $G_F/(\hbar c)^3 = (1.16639 \pm 0.00001) \times 10^{-5} \text{ GeV}^{-2}$, according to:

$$V_{ud}^2 = \frac{K}{2G_F^2(1 + \Delta_R^v)\overline{\mathcal{F}t}}. \quad (5)$$

With the nucleus-independent radiative correction adopted from Sirlin [7], $\Delta_R^v = (2.40 \pm 0.08)\%$, we obtain the result

$$|V_{ud}| = 0.9740 \pm 0.0005, \quad (6)$$

We are now in a position to test the unitarity of the CKM matrix by evaluating the sum of squares of the elements in its first row. With the value just obtained for V_{ud} , combined with the values of V_{us} and V_{ub} quoted by the Particle Data Group [2], the unitarity sum becomes

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9968 \pm 0.0014, \quad (7)$$

which differs from unity at the 98% confidence level.

Although neutron decay does not yet afford the precision possible with the superallowed decays, it does yield an independent result for V_{ud} . On the one hand, free neutron decay has an advantage over nuclear decays since there are no nuclear-structure dependent corrections to be calculated. On the

other hand, it has the disadvantage that it is not purely vector-like but has a mix of vector and axial-vector contributions. Thus, in addition to a lifetime measurement, a correlation experiment is also required to separate the vector and axial-vector pieces. Both types of experiment present serious experimental challenges. A recent survey [1] of world data on neutron decay, when augmented by a newly published measurement of the beta asymmetry in neutron decay [8], yields a value for the CKM matrix element of $|V_{ud}| = 0.9740 \pm 0.0017$. This value agrees exactly with the result from the superallowed decays, Eq. (6), but carries an uncertainty that is more than three times larger. With its relatively large uncertainty, the neutron result is consistent both with the nuclear result and with unitarity.

3 How significant is apparent non-unitarity?

The result in equation (7) is a very provocative one. If it is taken at face value, it indicates the need for some extension to the electroweak Standard Model, possibly indicating the presence of right-hand currents or of a scalar interaction [1]. This would have profound implications. However, the result could have a more trivial explanation. It could instead reflect some undiagnosed inadequacy in the calculated radiative or Coulomb corrections used to evaluate V_{ud} – or possibly a comparable inadequacy in the evaluation of V_{us} . What can be stated with some certainty is that the experimental results for the nine nuclei contributing to Fig. 1 cannot be at fault. Not only do they originate from a large number of independent measurements, but also the error bar associated with $|V_{ud}|$ is *not* predominantly experimental in origin. In fact, if experiment were the sole contributor, the uncertainty would be only ± 0.0001 . The largest contributions to the $|V_{ud}|$ error bar come from Δ_R^V (± 0.0004) and δ_C (± 0.0003).

Thus, if we are to determine whether the minimal Standard Model has failed, we must eliminate all possible “trivial” explanations for the apparent non-unitarity. To do so, nuclear physicists focus on the reliability of the calculated corrections in V_{ud} . (Others are re-evaluating V_{us} – see Ref. [1].) But, if there is a fault in the corrections, what size effect are we seeking? To restore unitarity, the calculated radiative corrections (δ_R or Δ_R^V) for all nine superallowed transitions would all have to be shifted downwards by 0.3%, or the calculated Coulomb corrections, δ_C , all shifted upwards by 0.3%, or some combination of the two. Such changes would constitute a substantial fraction of the total values of these small quantities. We have recently re-examined [1] the calculation of the various correction terms and conclude that such large changes are very implausible, particularly in the case of δ_R , the calculation of which involves standard QED and is well verified.

Even δ_C is well substantiated by several independent, yet concordant calculations [5, 6, 9, 10] and by measurements [11, 12] of the weak non-analog $0^+ \rightarrow 0^+$ β -transitions from ^{38m}K , ^{42}Sc , ^{46}V , ^{50}Mn and ^{54}Co . Such transitions can only occur via admixed components from the analog-state wave function, and are a sensitive test of charge-dependent mixing in five of the very nuclei whose superallowed branches contribute to the determination of V_{ud} . In all cases, the calculations used to obtain δ_C also yielded values for the non-analog transitions that agree very closely with experiment [13]. Thus, we must conclude that δ_C , like δ_R , is firmly based, and unlikely to conceal any fault that could shift all of its values substantially outside their quoted error bars. It is the reduction of those error bars that should preferably be the goal of future experiments.

4 Opportunities for the future

Given the importance of their contribution to the uncertainty now quoted for V_{ud} , it is the precision of Δ_R^V and δ_C that must be increased if the unitarity test is to be improved substantially. Indeed, Δ_R^V is inherent to the determination of V_{ud} whether the latter is obtained from neutron decay or from the superallowed transitions and, until the calculation of Δ_R^V is improved, its relative imprecision will continue to limit the unitarity test at about the present level regardless of any other improvements, experimental or theoretical. Improved calculations of Δ_R^V must therefore take priority as the most critical task for the future. This appears to lie entirely within the realm of theory: there have been no suggestions for experiments that could help to refine existing calculations or to confirm new ones.

The charge-dependent correction, δ_C , is a different story. Theoretical improvements are certainly possible, but experiment can provide independent controls on the calculations which, if successful, will reduce the uncertainty of the results. On the theoretical side, a large-basis (no core) shell model has already been used [10] to calculate δ_C for the ^{10}C superallowed transition. The extension of such detailed calculations to heavier nuclei will make important additions to the results of the more phenomenological methods used to date. Experimentally, there are three different approaches currently being followed: 1) increasing experimental precision on the nine known ft -values; 2) measuring new $0^+ \rightarrow 0^+$ decays from $T_z = 0$ nuclei with $A \geq 62$; and 3) measuring new $0^+ \rightarrow 0^+$ decays from $T_z = -1$ nuclei with $18 \leq A \leq 42$.

At first glance, experiments seeking to improve the measured ft -values for the nine superallowed transitions, whose small error bars already contribute very little to the uncertainty in V_{ud} , might seem to be a fruitless endeavor. Certainly, considering the large quantity of careful measurements now contributing to the content of Fig. 1, there is little chance that the central value of $\overline{\mathcal{F}t}$ will be changed significantly by a few more. However, the test of CVC can definitely be made more demanding as the experimental precision is increased and, to the extent that the $\mathcal{F}t$ -values continue to agree with one another, this would demonstrate at the same time the reliability of the δ_C calculations, which compensate for the transition-to-transition variations evident in the uncorrected ft -values. Of course, it is only the *relative* values of δ_C that can be tested by this method, but it would be a pathological fault indeed that could calculate in detail the required variations in δ_C while failing to obtain their *absolute* values to comparable precision.

The other experimental approaches aim to increase the number of superallowed emitters submitted to precision studies, either by adding new heavy odd-odd $T_z = 0$ cases, like ^{62}Ga and ^{74}Rb , or new light even-even $T_z = -1$ ones, like ^{22}Mg , ^{30}S and ^{34}Ar . Each region has its advantages and disadvantages as outlined in Table 1. Both regions are attractive to tests of δ_C because that correction is predicted [5, 6] to take larger values than the 0.2 – 0.6% range covered by the transitions shown in Fig. 1. Unfortunately, among the $A \geq 62$ nuclei there is not yet a reliable nuclear model with which to calculate an accurate δ_C ; even if experiment showed significant disagreements with current predictions, it is not clear what implications that would have for the nine transitions whose ft -values are now used to determine V_{ud} . The δ_C corrections for the latter transitions have been made in a completely different model space, where shell-model calculations have been extensively refined and tested on a wide variety of nuclei and properties. In contrast, the $T_z = -1$ nuclei between $A = 18$ and 42, overlap exactly the same well known model space as the transitions in Fig. 1: any observed discrepancies with experiment will reflect directly on the current determi-

Table 1: Present status of new regions of $0^+ \rightarrow 0^+$ superallowed emitters.

| Property | $A \geq 62$ | $18 \leq A \leq 42$ |
|---|-------------|---------------------|
| | $T_z = 0$ | $T_z = -1$ |
| large δ_C values? | $\sim 2\%$ | 0.3% – 1.2% |
| reliable nuclear model for δ_C ? | no | yes |
| IMME information available? | no | yes |
| 0^+ states available in daughter? | yes | likely not |
| 0^+ states known in daughter? | no | some |
| half-lives ≥ 0.5 s? | no | yes |
| reasonable production rate? | for some | yes |
| competing branches? | many, weak | few, strong |

nation of V_{ud} . Although the predicted values of δ_C for these nuclei are not as large as for those with $A \geq 62$, some are substantially larger than for the nine transitions currently known. Together, these two features make them very attractive cases for immediate experimental investigation.

In the longer term, however, the $A \geq 62$ nuclei should also provide valuable information about δ_C – as well as presenting an exciting experimental challenge, especially for this audience, which is already predisposed towards the fertile ground around $N = Z$ nuclei. A real challenge it is though! Not only must the superallowed decay branches themselves be studied but other pieces of experimental information have to be assembled. To arrive at a reliable nuclear model with which to calculate δ_C , some basic knowledge at least of low-lying excited states – their energies and spin-parities – is required for nuclei in the region of interest. To tune the calculation of δ_C , it is essential to know the coefficients of the IMME for the 0^+ multiplet involved in the superallowed transition. In lighter nuclei, all three members of the $T = 1$ multiplets are known and their masses measured; for the cases with $A \geq 62$, the $T_z = -1$ nuclei are, as yet, undiscovered and their adequate production with existing accelerators is, at best, problematic.

To test the efficacy of δ_C calculations, it is also valuable to know the location of nearby 0^+ states and to measure the weak, non-analog β -transitions that feed them. As the table indicates, neither region of new superallowed emitters offers this advantages for the time being. For the lighter nuclei, some excited 0^+ states are known but neither these nor any others that may be found in future are likely to be energetically available to β -decay. This limitation does not persist for the $A \geq 62$ emitters, whose β -decay energy windows are considerably larger, as is shown in Fig. 2. Clearly, there is great potential to study non-analog $0^+ \rightarrow 0^+$ transitions in these nuclei and several studies are currently underway on ^{62}Ga and ^{74}Rb decays. However, all such studies are hampered by the absence of any information on the location of 0^+ states in the daughters. It is very difficult to characterize β -transitions at the parts-per-million level without knowing in advance exactly how the daughter states decay. Identification of 0^+ states is usually best accomplished via two-nucleon transfer reactions but, except for ^{62}Zn , none of the daughter states in this region is readily accessible from a stable target. Widespread measurements will undoubtedly have to await the availability of intense radioactive beams.

The last three lines in Table 1 address the fundamental issues that experiments will face in making ft -value measurements at a level of precision to match existing data on $0^+ \rightarrow 0^+$ superallowed

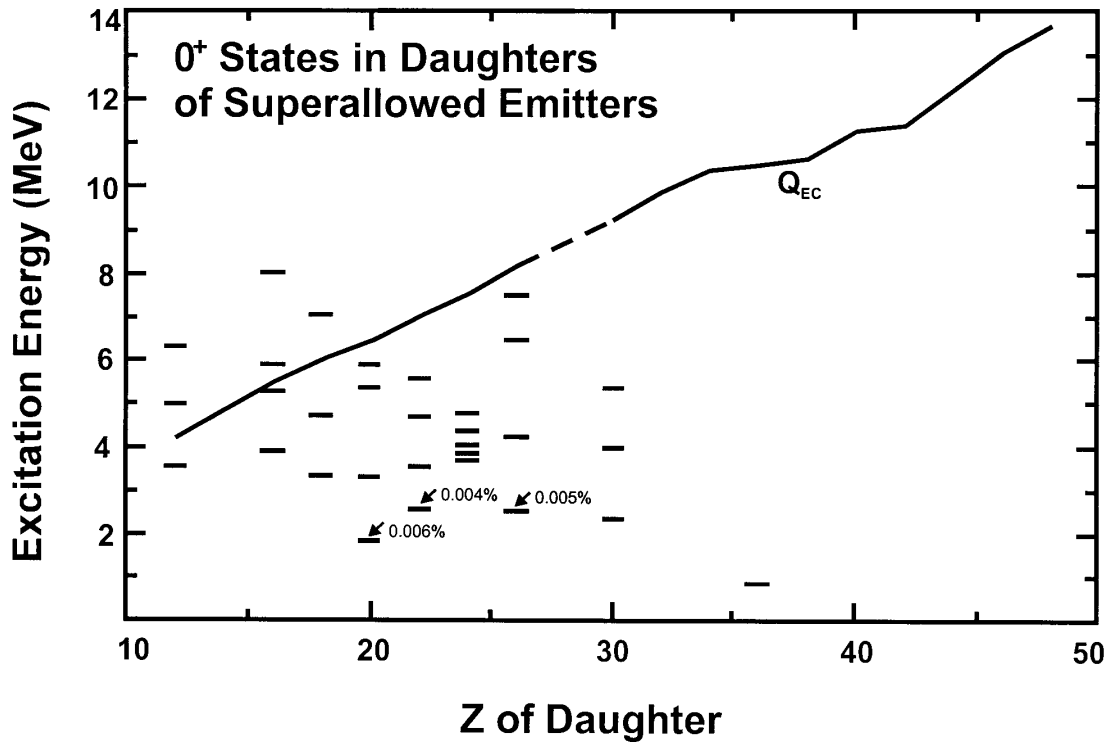


Figure 2: Location of known excited 0^+ states in the daughters of $T_z = 0$ superallowed emitters, together the Q_{EC} window open to β -decay. The measured branching ratio (as a percent) is shown for the only three cases where the β -branch to the excited 0^+ state has been measured.

decays. First, the masses of both parent and daughter will have to be measured to a precision of 1×10^{-8} . Even on-line Penning traps are not currently foreseen to achieve this level of precision for activities with half-lives less than 0.5 s. Second, the half-life itself must be determined to about five parts in 10^4 ; for this, repeated low-background measurements, each including at least a million counts, must be performed. The production rate of the activity must obviously match this requirement. Finally, where other β -decay branches compete with the superallowed one, their intensity must be carefully measured so that the superallowed branching ratio can be established with a precision of better than 1×10^{-3} . This is a difficult task for both regions of new emitters, the one because there could be a significant number of individually weak transitions, the other because the few transitions that do occur (to 1^+ states) are large and must be measured with challenging precision.

5 Conclusions

I hope I have convinced you that current data strongly suggest a failure of the minimal Electroweak Standard Model. With nuclear data playing a key role, the CKM matrix has been shown to miss unitarity by more than two standard deviations. The quest to make this unitarity test more definitive stands as one of the most important challenges open to nuclear physicists in the future. The required measurements all lie in the region of $N = Z$ nuclei and encompass much more than β -decay studies. Energy-level spectroscopy, transfer reactions and mass measurements are also

required, as are calculations of nuclear models and improved radiative corrections. The prospects are exciting.

Acknowledgements

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